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Decay Constants and Semileptonic Form Factors of Pseudoscalar Mesons[1]

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Abstract

A relativistic constituent quark model is adopted to give an unified description of the leptonic and semileptonic decays of pseudoscalar mesons (π, K, D, D_s, B, B_s). The calculated leptonic decay constants and form factors are found to be in good agreement with available experimental data and the results of other approaches. Eventually, the model is found to reproduce the scaling behaviours of spin-flavor symmetry in the heavy-quark limit.

1 Introduction

Semileptonic decays of pseudoscalar mesons allow to evaluate the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, which are fundamental parameters of the Standard Model. The decay $K \to \pi e\nu$ provides the most accurate determination of V_{us}, the semileptonic decays of D and B mesons, $D \to K(K^*)l\nu$, $B \to D(D^*)l\nu$ and $B \to \pi(\rho)l\nu$, can be used to determine $|V_{cs}|$, $|V_{cb}|$ and $|V_{ub}|$, respectively. This program can be performed if the non-perturbative QCD effects, which are parameterized by the form factors, are known. Up to now, these form factors cannot be evaluated from first principles, thus models, more or less connected with QCD, are usually considered for this purpose. Here we discuss a relativistic quark model [1], previously used to study the baryon form factors [2].

This model is based on an effective Lagrangian describing the coupling of mesons with their constituent quarks. The physical processes are described by the one-loop quark diagrams and meson-quark vertices related to the Bethe-Salpeter amplitudes. In principle, the vertex functions and quark propagators should be given by the Bethe-Salpeter and Dyson-Schwinger equations, respectively. This kind of analysis is provided by the Dyson-Schwinger Equation (DSE) studies [3] and an unified description of light and heavy meson observables was carried out in [4, 5]. Here, instead, we use free propagators for constituent quarks and consider a Gaussian vertex function as Bethe-Salpeter confining function. The adjustable parameters, the widths of Bethe-Salpeter amplitudes in momentum space, and the constituent quark masses, are determined from the best fit of available experimental data and some lattice simulations. Our results are in good agreement with experimental data and other approaches. We also reproduce the spin-flavor symmetry relations and scaling for leptonic decay constants and semileptonic form factors in the heavy-quark limit [6].

An approach similar to the one presented here, can be found in [7]. That model is based on meson-quark interactions, so the mesonic transition amplitudes are described by diagrams with heavy mesons attached to quark loops. The free propagator has been used for light quarks, while the quark propagator obtained in the heavy-quark limit has been adopted for heavy quarks.

2 Our model

Our starting point is the effective Lagrangian describing the coupling between hadrons and quarks. The

$$\mathcal{L}_{\rm int}(x) = g_H H(x) \int dx_1 \int dx_2 \Phi_H(x; x_1, x_2) \bar{q}(x_1) \Gamma_H \lambda_H q(x_2) \tag{1}$$

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describes the transition of the meson H (λ_H is the corresponding combination of Gell-Mann matrices, Γ_H are Dirac matrices) into its constituents q_1 and q_2. The function Φ_H is related to the scalar part of the Bethe–Salpeter amplitude.

The coupling constant g_H is given by the derivative of the meson mass operator $\widetilde{\Pi}_H$ by the *compositeness condition* [8]:

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2}\widetilde{\Pi}'_H(m_H^2) = 0\,. \tag{2}$$

It is worth noticing that, due to the absence of confinement, the sum of constituent quark masses should be larger than the mass of the corresponding meson otherwise, imaginary parts in physical quantities appear. This allows us to consider low-lying pseudoscalar mesons only.

Now, to give an example of the hadronic part of invariant amplitudes, we will evaluate the form factor f_+, which appears in the semileptonic decays of a pseudoscalar meson into another one, $H \to H'\ell\nu$. The invariant amplitude can be written as:

$$A(H(p) \to H'(p')\, e\, \nu) = \frac{G_F\, V_{q_1 q_2}}{\sqrt{2}}\left[\bar{\ell}\,\gamma_\mu(1-\gamma_5)\,\nu\right] M^\mu_{HH'}(p,p'), \tag{3}$$

where G_F is the Fermi constant, $V_{q_1 q_2}$ is the corresponding Cabibbo–Kobayashi–Maskawa matrix element, and, in our model, $M^\mu_{HH'}(p,p')$ is given by:

$$\begin{aligned} M^\mu_{HH'}(p,p') &= \frac{3\,g_H\,g_{H'}}{4\,\pi^2}\int\frac{d^4k}{4\pi^2 i}\phi_H(-k^2)\phi_{H'}(-k^2)\mathrm{tr}\Big[\gamma^5 S_3(\not k)\gamma^5 S_2(\not k+\not p')\gamma^\mu(1-\gamma_5)S_1(\not k+\not p)\Big] \quad (4)\\ &= \frac{3\,g_H\,g_{H'}}{4\,\pi^2}\Big[I_+(p^2,p'^2,q^2)(p+p')^\mu + I_-(p^2,p'^2,q^2)q^\mu\Big]\\ &= f_+(q^2)(p+p')^\mu + f_-(q^2)q^\mu\,. \end{aligned}$$

Here, $q = p - p'$, $S_i(k) = 1/(m_i - \not k)$ is the propagator of the quark i with mass m_i.

To evaluate the integral in Eq. (4) we have to calculate the following integrals ($\mathcal{F}(-k^2) = \phi_H(-k^2)\cdot\phi_{H'}(-k^2)$):

$$J^{(0,\mu,\mu\nu,\mu\nu\delta)} \equiv \int\frac{d^4k}{\pi^2 i}\frac{(1,k^\mu,k^\mu k^\nu,k^\mu k^\nu k^\delta)\mathcal{F}(-k^2)}{[m_1^2-(k+p)^2][m_2^2-(k+p')^2][m_3^2-k^2]}\,. \tag{5}$$

They can be evaluated using the standard Feynman α-representation and the integral Cauchy representation for the function $\mathcal{F}$:

$$J^0 = \int\limits_0^\infty dt\left(\frac{t}{1+t}\right)^2\int d^3\alpha\,\delta\left(1-\sum_{i=1}^3\alpha_i\right)\left(-\frac{d\mathcal{F}(z_I)}{dz_I}\right) \tag{6}$$

$$J^\mu = -\int\limits_0^\infty dt\left(\frac{t}{1+t}\right)^3\int d^3\alpha\,\delta\left(1-\sum_{i=1}^3\alpha_i\right)P^\mu\left(-\frac{d\mathcal{F}(z_I)}{dz_I}\right) \tag{7}$$

$$J^{\mu\nu} = \int\limits_0^\infty dt\left(\frac{t}{1+t}\right)^2\int d^3\alpha\,\delta\left(1-\sum_{i=1}^3\alpha_i\right)\left\{-\frac{1}{2}g^{\mu\nu}\frac{1}{1+t}\mathcal{F}(z_I) - P^\mu P^\nu\left(\frac{t}{1+t}\right)^2\left(-\frac{d\mathcal{F}(z_I)}{dz_I}\right)\right\} \tag{8}$$

$$\begin{aligned} J^{\mu\nu\delta} = \int\limits_0^\infty dt\left(\frac{t}{1+t}\right)^2\int d^3\alpha\,\delta\left(1-\sum_{i=1}^3\alpha_i\right)\left\{\frac{1}{2}\Big[g^{\mu\nu}P^\delta + g^{\mu\delta}P^\nu + g^{\nu\delta}P^\mu\Big]\frac{t}{(1+t)^2}\mathcal{F}(z_I)\; +\right.\\ \left. P^\mu P^\nu P^\delta\left(\frac{t}{1+t}\right)^3\left(-\frac{d\mathcal{F}(z_I)}{dz_I}\right)\right\} \end{aligned} \tag{9}$$

where $P = \alpha_1 p + \alpha_2 p'$, $D_3 = \alpha_1\alpha_3 p^2 + \alpha_2\alpha_3 p'^2 + \alpha_1\alpha_2 q^2$, and

$$z_I = t\left[\sum_{i=1}^{3}\alpha_i m_i^2 - D_3\right] - \frac{t}{(1+t)}\,P^2. \tag{10}$$

Finally, we have the analytical expression for I_+

$$\begin{aligned} I_+(p^2,p'^2,q^2) &= \frac{1}{2}\int\limits_0^\infty dt\left(\frac{t}{1+t}\right)^2\int d^3\alpha\,\delta\left(1-\sum_{i=1}^{3}\alpha_i\right)\left\{\mathcal{F}(z_I)\frac{1}{1+t}\left[4-3(\alpha_1+\alpha_2)\frac{t}{1+t}\right]\right. \\ &\quad +\left(-\frac{d\mathcal{F}(z_I)}{dz_I}\right)\left[(m_1+m_2)m_3+\frac{t}{1+t}\left(-(\alpha_1+\alpha_2)(m_1m_3+m_2m_3-m_1m_2)\right.\right. \\ &\quad \left.\left.\left.+\alpha_1 p^2+\alpha_2 p'^2\right)-P^2\left(\frac{t}{1+t}\right)^2\left(2-(\alpha_1+\alpha_2)\frac{t}{1+t}\right)\right]\right\}, \end{aligned} \tag{11}$$

which can be used also to obtain the normalization constants g_H and $g_{H'}$ in Eq. (2). For example, g_H is given by

$$g_H = \sqrt{\frac{4\pi^2}{3I_+(p^2,p^2,0)}} \tag{12}$$

where we put $m_1 = m_2 \equiv m$.

3 Results and discussion

Once obtained the analytical expressions for the invariant amplitudes, a comparison with the experimental data is in order. For this purpose we have to specify the analytical form of $\phi_H(-k^2)$ and the constituent masses appearing in the expressions. In particular, we choose the Gaussian form $\phi(-k^2) = \exp\{k^2/\Lambda_H^2\}$ in Minkowski space, where the magnitude of Λ_H characterizes the size of the BS-amplitude and is an adjustable parameter in our approach. Thus, to describe processes involving π, K, D, D_s, B, and B_s mesons, we have six Λ-parameters plus four quark masses ($m_q = m_u = m_d$, m_s, m_c, and m_b), all of them are fixed *via* the least-squares fit to the observables measured experimentally or taken from lattice simulations (as is reported in Table 1).

Table 1: Calculated values of a range of observables ($g_{\pi\gamma\gamma}$ in GeV^{-1}, leptonic decay constants in GeV, form factors and ratios are dimensionless). The "Obs." are extracted from Refs. [9, 10, 11, 12, 13, 14, 15]. The quantities used in fitting free parameters are marked by "$*$".

		Obs.	Calc.			Obs.	Calc.
$*$	$g_{\pi\gamma\gamma}$	0.274	0.242		$f_+^{K\pi}(0)$	0.98	0.98
$*$	f_π	0.131	0.131	$*$	$f_+^{DK}(0)$	0.74 ± 0.03	0.74
$*$	f_K	0.160	0.160		$f_+^{BD}(0)$		0.73
$*$	f_D	0.191^{+19}_{-28}	0.191		$f_+^{B\pi}(0)$	0.27 ± 0.11	0.51
$*$	$\frac{f_{D_s}}{f_D}$	1.08(8)	1.08		$\mathrm{Br}(K\to\pi l\nu)$	$(4.82\pm0.06)\cdot10^{-2}$	$4.4\cdot10^{-2}$
	f_{D_s}	0.206^{+18}_{-28}	0.206		$\mathrm{Br}(D\to Kl\nu)$	$(6.8\pm0.8)\cdot10^{-2}$	$8.1\cdot10^{-2}$
$*$	f_B	0.172^{+27}_{-31}	0.172		$\mathrm{Br}(B\to Dl\nu)$	$(2.00\pm0.25)\cdot10^{-2}$	$2.3\cdot10^{-2}$
$*$	$\frac{f_{B_s}}{f_B}$	1.14(8)	1.14		$\mathrm{Br}(B\to\pi l\nu)$	$(1.8\pm0.6)\cdot10^{-4}$	$2.1\cdot10^{-4}$
	f_{B_s}		0.196				

The fitted values for the free parameter of our model are the following:

$$(\Lambda_\pi,\, \Lambda_K,\, \Lambda_D,\, \Lambda_{D_s},\, \Lambda_B,\, \Lambda_{B_s}) \quad = \quad (1.16,\, 1.82,\, 1.87,\, 1.95,\, 2.16,\, 2.27)\; GeV \tag{13}$$

$$(m_q,\, m_s,\, m_c,\, m_b) \quad = \quad (0.235,\, 0.333,\, 1.67,\, 5.06)\; GeV\,. \tag{14}$$

Note that the Λ values are larger for mesons with larger mass, *i.e.* $\Lambda_H < \Lambda_{H'}$ when $m_H < m_{H'}$. This correctly corresponds to the ordering law for the sizes of bound states.

The u(d)-quark mass and the parameter Λ_π are almost fixed from the rate $\pi \to \mu\nu$ and $\pi^0 \to \gamma\gamma$ with an accuracy of a few percent. Moreover, the obtained value of m_u is less than the constituent-light-quark mass used in baryon physics.

Let us now consider the q^2-behaviour of the form factors. Since a numerical integration should be done (see, Eq. (11)), we do not have a simple analytical expression for them. However, looking at the numerical results, the form

$$f_+^{HH'}(q^2) = \frac{f(0)}{1 - b_0\left(\dfrac{q^2}{m_H^2}\right) - b_1\left(\dfrac{q^2}{m_H^2}\right)^2} \tag{15}$$

is suitable for a good description of the q^2-behaviour, once the parameters b_0, b_1 and $f(0)$ are fixed. Their values are collected in the following Table:

	$K\to\pi$	$D\to K$	$B\to D$	$B\to\pi$
$f(0)$	0.98	0.64	0.73	0.51
b_0	0.28	0.64	0.77	0.52
b_1	0.057	0.20	0.19	0.38

(16)

It should be noted that a value for $f_+^{B\pi}(0)$ larger than those obtained by QCD Sum Rules [16], and other approaches [18]. In any case, as one can see from the Table 1, the agreement between our predictions and experimental data on semileptonic branching ratios is impressive.

As we have seen, our model gives an accurate and unified description of the weak and radiative ($\pi^0 \to \gamma\gamma$) transitions involving pseudoscalar mesons. Moreover, as already stated in the introduction, it is able to reproduce the scaling behavior predicted by QCD in the heavy-quark limit. For more details, see the original paper. Here we report the way to obtain this limit in the expression for f_+. The heavy-quark limit corresponds to consider $m_1 \equiv M \to \infty$, $m_2 \equiv M' \to \infty$ and $p^2 = (M+E)^2$, $p'^2 = (M'+E)^2$ with E being a constant value independent M and M'. By replacing in Eq. (11) the variables α_1 with α_1/M and α_2 with α_2/M', one obtains

$$\begin{aligned} I_+ \quad &\to \quad \frac{M+M'}{2MM'}\cdot \int_0^\infty dt \left(\frac{t}{1+t}\right)^2 \int_0^1 d\alpha\alpha \int_0^1 d\tau \Big(-\mathcal{F}'(z)\Big)\left[m + \frac{\alpha t}{1+t}\right] \\ &= \quad \frac{M+M'}{2MM'}\cdot \frac{1}{2}\int_0^1 \frac{d\tau}{W}\int_0^\infty du \mathcal{F}(\tilde{z})\frac{m+\sqrt{u/W}}{m^2+\tilde{z}} \end{aligned} \tag{17}$$

where

$$\tilde{z} = u - 2E\sqrt{\frac{u}{W}}, \qquad W = 1 + 2\tau(1-\tau)(w-1), \qquad w = \frac{M^2+M'^2-q^2}{2MM'}\,. \tag{18}$$

Therefore, using the relation between I_+ and f_+ and the normalization condition, the correct scaling relation is found:

$$f_+ \to \frac{M' + M}{2\sqrt{MM'}}\xi(w) \qquad \xi(w) \propto \int_0^1 \frac{d\tau}{W}\int_0^\infty du\, \phi_H^2(\tilde{z})\,\frac{m+\sqrt{u/W}}{m^2+\tilde{z}}\,. \tag{19}$$

In conclusion, we can see that the agreement with experimental data and lattice results is very good, with the exception of the value of $f_+^{bu}(0)$ which is found to be larger than the monopole extrapolation of a lattice simulation, QCD Sum Rules (cf. [16]) and some other quark models (see, for example, [17, 18]). However, this result is consistent with the value calculated from Refs. [5, 19] and a light-front constituent quark model [20]. Moreover, it allows us to reproduce the experimental data on $B \to \pi l \nu$ decay.

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